

# Exotic Hadrons and Underlying $Z_{2,3}$ Symmetries

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September 22, 2015

## Abstract

The recent observation of higher quark combinations, tetraquarks and pentaquarks, is a strong indication of more exotic hadrons. Using  $Z_2$  and  $Z_3$  symmetries and standard model data, a general quark combination producing new hadronic states is proposed in terms of polygon geometries according to the Dynkin diagrams of  $\hat{A}_n$  affine Lie algebras. It has been shown that  $Z_{2,3}$  invariance is crucial in the determination of the mesonic or the baryonic nature of these states. The hexagonal geometry is considered in some details producing both mesonic and baryonic states. A general class of this family is also presented.

**KeyWords:** Standard Model, Exotic Hadrons and Lie symmetries.

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For a long time, it has been realized that the experimental situation in the Quantum Chromodynamics (QCD) is quite stable and well understood [1, 2]. This describes the interaction between quarks carrying the color charge of the strong interaction by force mediating gluons which can not only couple to quarks but also to themselves, due to the  $SU(3)_C$  gauge theory. They carry color charge. Although  $SU(3)_C$  describes the strong interactions of particles, until today experiments have not been able to detect colored objects directly. However, only colorless hadrons can be observed. The explanation of this fact and the combination of hadrons from quarks and gluons remain incomplete.

Understanding how matter is formed is one of the big problem in modern particle physics. It is known that the biggest part of the observable matter is made of strongly interacting quarks and gluons. The standard model (SM) of particle physics constitutes one of the most succeeded realizations in this field. Based on the SM symmetry group  $SU(3)_C \times SU(2)_L \times U(1)_Y$  describing strong, weak and electromagnetic interactions, it provides an elegant theoretical framework which is able to describe with a high precision the known experimental phenomenon in modern physics [3, 4]. However, with the progressive probes of high energies in experiments and results become more precise, the SM satisfactory is still far and physics beyond is widely expected to reside with new elementary particles, hadrons, and symmetries [5, 6].

The richness of the hadronic spectrum could be revealed by a fast look into the particle spectrum [7]. The large number of the hadronic states has clearly suggested the existence of a deeper level where the messy hadronic world can be easily understood in terms of a few constituent of spin- 1/2 quark flavors [8]. At this level, in agreement with the QCD, almost all observed states could be clearly identified as two- or three-quark states. More precisely, quark-antiquark  $q\bar{q}$  are known as mesons and three quark states  $qqq$  are known as baryons. In this picture, the entire hadronic spectrum could be nicely classified as the colorless mesons and baryons.

In the last years, results in the strong sector  $SU(3)_C$  have opened new windows in the understanding of flavor physics including quark configurations [8, 9, 10, 11]. In particular, it has been suggested some four quark states. This suggestion has been supported by an observation of new resonances corresponding to four quark states around 2-5  $GeV$ . They belong to a strange series of particles known by  $X$ ,  $Y$  and  $Z$  states. They were named  $X$ ,  $Y$ ,  $Z$  [12, 13, 14, 15]. Pentaquark states discovery was also suggested many years ago by several experiments. However, due to poor data and statistical analysis such claims were not accepted. Very recently, new results consistent with pentaquark states was reported by the LHCb collaboration at CERN [16]. Decay experiments are expected to run soon in order to determine its nature with more precision. Motivated by these recent works, there is no reason to ignore higher quark combinations.

In this work, we aim to give a possible generalization dealing with hadronic states using a geometric approach based on discrete symmetries derived from the SM gauge group. More

precisely, the centers of the  $SU(3)$  and  $SU(2)$  Lie symmetries of the SM will guide us to specify the possible orders for the generalization. To keep contact with the observed physics, we present a general quark configuration of the mesonic and baryonic states exhibiting  $Z_2$  and  $Z_3$  global geometrical symmetries. In fact, these symmetries have been not explored in the related physics. It does not mean that there have no roles in modern particle physics. Here, we show up their importance. In particular, they allow one to propose possible higher quark states beyond the usual hadrons known in the SM.

A close inspection shows that the  $Z_2$  and  $Z_3$  symmetries can produce a higher combination of quarks associated with exotic hadronic states. To do so, it is convenient to use the following notation,

$$M : |q\bar{q}\rangle, \quad B : |qqq\rangle. \quad (1)$$

It is observed that these states are invariant under  $Z_2$  and  $Z_3$  symmetries respectively. The mesonic states containing  $q$  and  $\bar{q}$  are invariant under the  $Z_2$  symmetry acting as,

$$q \rightarrow -q, \quad \bar{q} \rightarrow -\bar{q}. \quad (2)$$

However, the baryonic states are invariant under the  $Z_3$  symmetry acting as

$$q \rightarrow wq, \quad w^3 = 1. \quad (3)$$

It is important to recall that the quadratic combination  $|qq\rangle$  is invariant under  $Z_2$  acting only as  $q \rightarrow -q$ . However, it is not allowed by the confinement phenomenon<sup>1</sup>. This kind of states and their generalization should be projected out.

According to the SM data and the  $Z_{2,3}$  symmetries, we give an extended combination for the allowed hadronic states. To be precise, we explore techniques of polygon geometries to support the existence of such exotic states. Our analysis will be based on geometric shapes which are invariant under the above mentioned symmetries. In fact, each hadronic state will correspond to an allowed polygon geometry playing similar role as graph theory used to encode physical data including gauge and matter fields derived from string theory compactification using quiver method [18]. In this scenario, the  $Z_2$  and  $Z_3$  symmetries will be crucial in the determination of the nature of such states. To show how the philosophy works in practice, let us first start by giving the two leading polygon geometry pieces playing a primordial role in the building block of higher hadronic states. In this way, the mesonic state (1) is represented by two connected nodes, and the leading baryonic state  $|qqq\rangle$  is represented by three nodes forming a triangle. At first sight, it seems that these graphs share similarities with the toric geometry realization of complex manifolds[19]. However, the SM Lie symmetries push us to think about polygon

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<sup>1</sup>Only colorless hadrons can be observed.

geometries associated with such symmetries. Considering closed quiver diagrams and taking into account of  $Z_{2,3}$  symmetries, the  $su(2)$  and  $su(3)$  Lie symmetries should be shown up in the discussion. An investigation reveals that these quivers can be identified with the corresponding affine Dynkin geometries [20]. This is illustrated in figure 1. Inspired by the recent theoretical



Figure 1: Standard hadrons: mesons (left) and baryons (right).

and experimental works [10, 11, 17], the extended hadronic states will be represented by closed Dynkin diagrams associated with the Affine  $\hat{A}_n$  series. This geometric approach based on the  $Z_2$  and  $Z_3$  symmetries is interesting since one can control completely the engineering of allowed diagrams associated with possible hadronic states.

Naturally, the first extended polygon geometry will be associated with four nodes corresponding to the  $\hat{A}_3$  affine Dynkin diagram. A priori, there are many  $q$  and  $\bar{q}$  systems which can be placed on these four nodes. However, the above physical requirement allow only one possible configuration invariant under a geometric  $Z_2$  leading to a mesonic state  $|q\bar{q}q\bar{q}\rangle$ . Such states have been investigated in [10, 11], and have been supported by the recent experimental result [17]. Roughly, the corresponding diagram and the geometric  $Z_2$  symmetry are represented in the figure 2. With such four quarks constituents, several ways of combinations are possible.

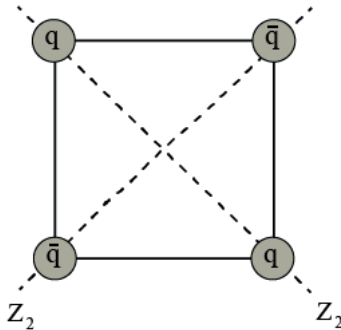


Figure 2: Tetraquark mesons.

Indeed, a tetraquark can either contain four quarks  $|q\bar{q}q\bar{q}\rangle$  or something resembling two ordinary

mesons bound together  $|(q\bar{q}), (q\bar{q})\rangle$  (a dimeson). Reported examples of such tetraquark states are the mesons  $X(3872)$ ,  $Y(4140)$ ,  $Z_c(3900)$  and  $Z(4430)^-$ . They differ by their flavors content  $|q\bar{q}q\bar{q}\rangle_X \neq |q\bar{q}q\bar{q}\rangle_Y \neq |q\bar{q}q\bar{q}\rangle_{Z_c} \neq |q\bar{q}q\bar{q}\rangle_Z$  which are still being under investigated to characterize their determined masses and quantum numbers. Concretely, the heaviest one,  $Z(4430)^-$ , with a negative charge -1 and a mass around 4,430 GeV, has been definitely identified with the tetraquark meson  $|q\bar{q}q\bar{q}\rangle \equiv |c\bar{c}d\bar{u}\rangle_Z$  [15]. It is worth noting that the  $|qqqq\rangle$ , for instance, is not allowed as required by the confinement phenomenon.

The next possible five quarks state  $|q\bar{q}qqq\rangle$  has been very recently also identified from the decay of the bottom lambda baryon  $\Lambda_b^0$  into a  $J/\Psi(c\bar{c})$  meson, a kaon  $K^-(s\bar{u})$  and a proton  $p(uud)$  by the LHCb with two hidden-charm pentaquarks as  $|q\bar{q}qqq\rangle \equiv |uud\bar{c}c\rangle_{P_C^+}$  being a charmonium-pentaquark [16]. Such state can be combined into two possible combinations: a five quark bag  $|q\bar{q}qqq\rangle$  an ordinary meson-baryon bounded together  $|(qqq), (q\bar{q})\rangle$ . It is interesting to note that, with respect to the  $Z_{2,3}$  symmetries, the latter combination is more likely.

After a graphic examination based on the  $Z_{2,3}$  invariance, the next fantastic example concerns six nodes associated with the hexagonal geometry. The appearance of this geometry is the more remarkable one since it appears naturally in many places in physics including even in the SU(3) gauge theory describing the strong interaction [21]. Indeed, the gluon gauge fields are associated with the root systems of the corresponding su(3) finite Lie symmetry. Its Weyl group is the symmetric group  $S_3 = D_3$  of order 6 which acts transitively and freely on the hexagon. It is obvious to see that this group contains  $Z_2$  and  $Z_3$  as subgroups. At this level, we remark that the presence of such symmetries seem to be useful in the present discussion. We believe that this feature would play an important role in the understanding of the hexagonal hadrons. In fact, we will see that this geometry produces both mesonic and baryonic states depending on the chosen symmetry. Later on, we show that this example constitutes in fact particular geometries of infinitely many possible quark configurations giving rise to both mesonic and baryonic states. For the hexagonal structure, we can obviously determine the nature of hadronic state by taking only one discrete symmetry. It is observed that the  $Z_3$  geometric invariance generates the baryonic state  $|qqqqqq\rangle$ . Its complex conjugate is also allowed giving the state  $|\bar{q}\bar{q}\bar{q}\bar{q}\bar{q}\bar{q}\rangle$ . However, the  $Z_2$  geometric invariance gives only a mesonic state denoted by  $|q\bar{q}q\bar{q}q\bar{q}\rangle$  in accordance with the known data. The corresponding diagrams are represented in the figure 3.

As for the four state quark constituents, such six quark ones involves several ways of combinations are possible. Indeed, a hexaquark can either contain six quarks  $|qqqqqq\rangle$  or something resembling two ordinary baryons bound together  $|(qqq), (qqq)\rangle$  a dibaryon, or three quarks and three antiquarks  $|(qqq), (\bar{q}\bar{q}\bar{q})\rangle$  a baryon antibaryon or six antiquarks  $|\bar{q}\bar{q}\bar{q}, (\bar{q}\bar{q}\bar{q})\rangle$  (a dian-tibaryon). Once formed, dibaryons are predicted to be fairly stable, i.e, the diproton and dineutron. The existence of a stable dibaryon having the light quark composition  $|qqqqqq\rangle = |udsuds\rangle$

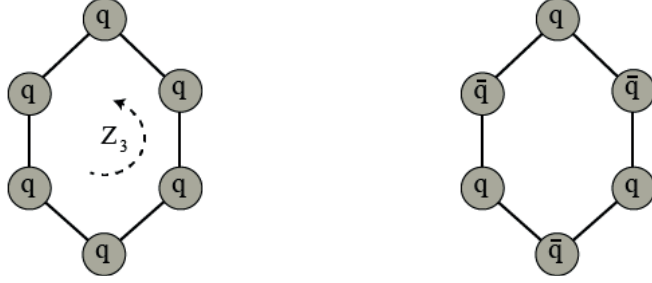


Figure 3: Hexaquark Baryons (left) and hexaquark mesons (right).

is possible. To detect such states, many experiments have been suggested for their decays and interactions where several candidates were observed but they are not yet confirmed [22]. The last potential one, called  $d^*(2380)$ , was detected at about  $2,380 \text{ GeV}$  for  $10^{-23} \text{ s}$  [23].

We can now generalize to higher geometries involving more than six nodes. To get a mesonic states, the geometry should consist of  $2+2n$  nodes according to the  $\hat{A}_{1+2n}$  Dynkin diagram. SM and the  $Z_2$  invariance require a system of  $(1+n)$  quarks and  $(1+n)$  antiquarks. However, the geometry corresponding to a baryonic state contains a collection of  $3+3m$  nodes according to the  $\hat{A}_{2+3m}$  affine Dynkin diagram. The corresponding states will be denoted by  $|q\bar{q} \dots q\bar{q} \dots q\bar{q}\rangle$  and  $|q \dots q \dots q\rangle$  respectively and are illustrated by the diagrams presented in figure 4.



Figure 4: Higher hadrons: higher baryons (left) and higher mesons (right).

It is worth noting to comment this generalization involving two interesting features. First, it is noted that the case  $n = m = 0$  corresponds to the standard baryonic and mesonic states. Second, it is observed that both the baryonic and mesonic states can appear when  $m = 2j + 1$  and  $n = 2m - j$  where  $j$  is a positive integer. This family of geometry goes beyond the case of the hexagon associated with  $j = 0$ .

In this work, we have proposed new quark combinations generating exotic hadronic states. The theoretical analysis has been relied on polygon geometries sharing similarities with the  $\hat{A}_n$  affine Dynkin diagrams. Inspired by the QCD description of the strong interaction and its

effective treatment of the quark states arising in standard pictures of the mesonic and baryonic states, we have explored the  $Z_{2,3}$  geometrical symmetries to construct possible higher quark states going beyond the standard hadrons. These symmetries have played an essential role for identifying the mesonic or the baryonic nature of the corresponding states. Although, we have tried to give a theoretical argument behind the existence of higher quark combinations, this present study deserves a deeper investigation since the observation of more exotic hadrons remains enhanced.

**Acknowledgments:** AB would like to thank his family in Témara (Morocco) for hospitality, where a part of the research reported in this work was conducted. He would like to thank also his mother Haja Fatima for all kind of supports.

## References

- [1] T. Muta, *Foundations of Quantum Chromodynamics*, Lect. Not. Phys-Vol. 5 (World Scientific, Singapore, 1987).
- [2] F.J. Yndurain, *The Theory of Quark and Gluon Interactions*, Texts and Monographs in Physics (Springer-Verlag, New York, 1999).
- [3] C. Quigg, *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions* (Benjamin- Cummings Publishing Company Co, New York, 1983).
- [4] T. P. Cheng and L. F. Li, *Gauge Theories of Elementary Particle Physics*, Oxford University Press, (London, 1984).
- [5] G. Bertone, D. Hooper, and J. Silk, *Particle Dark Matter: Evidence, Candidates and Constraints*, Phys. Rept. 405(2005)279-390.
- [6] S. Dimopoulos, S. Raby, and F. Wilczek, *Supersymmetry and the Scale of Unification*, Phys. Rev. **D24**(1981)1681.
- [7] C. Caso et al. (*Particle Data Group*), *Review of Particle Properties*, Eur.Phys. J. **C 3** (1998)1.
- [8] M. Gell-Mann, *A Schematic Model of Baryons and Mesons*, Phys. Lett. **8** (1964) 214.
- [9] H. Fritzsch and Z. Z. Xing, *Mass and Flavor Mixing Schemes of Quarks and Leptons*, Prog. Part. Nucl. Phys. **45** (2000)1-81.
- [10] S. Weinberg, *Pions in Large  $N$  Quantum Chromodynamics*, Phys.Rev.Lett. **105** (2010)261601.

- [11] S. Weinberg, *Tetraquark Mesons in Large  $N$  Quantum Chromodynamics*, Phys. Rev. Lett. **110** (2013)261601.
- [12] The LHCb collaboration, *Determination of the  $X(3872)$  meson quantum numbers*, Phys. Rev. Lett. **110**(2013), 222001.
- [13] G Cotugno, R Faccini, AD Polosa, C Sabelli, Charmed Baryonium, Phys.Rev.Lett.104:132005, (2010).
- [14] T. Xiao, S. Dobbs, A. Tomaradze, K.K Seth, Phys. Lett. **B 727** (2013) 366-370, [arXiv:1304.3036 \[hep-ex\]](#).
- [15] The LHCb collaboration, *Observation of the resonant character of the  $Z(4430)^-$  state*, Phys. Rev. Lett. **112** (2014) 222002.
- [16] The LHCb collaboration, [arXiv:1507.03414 \[hep-ex\]](#).
- [17] The LHCb collaboration, *Observation of the resonant character of the  $Z(4430)^-$  state*, Phys. Rev. Lett. **112** (2014) 222002.
- [18] A. Belhaj et al. *On fermion mass hierarchies in MSSM-like quiver models with stringy corrections*, CEJP **9**(2011)1458.
- [19] W. Fulton, *Introduction to Toric varieties*, Annals of Math. Studies, No .131, Princeton University Press, 1993.
- [20] V.G. Kac, *Infinite dimensional Lie algebras*, third edition, Cambridge University Press (1990).
- [21] M. Gell-Mann, Y. Neeman, *The Eightfold Way*, Benjamin, New York (1964).
- [22] KTeV Collaboration, *Search for the weak decay of a lightly bound  $H0$  dibaryon*. Phys. Rev. Lett. **84** (2000) 2593.
- [23] P. Adlarson et al., *Evidence for a New Resonance from Polarized Neutron-Proton Scattering*. Phys. Rev. Lett **112** (2014) 02301.